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Composite Model of Gauge Bosons in Electroweak Interactions

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We consider a simple Lagrangian which is constructed by only the preon and antipreon fields. By introducing the auxiliary fields ϕ_μ, $\phi_\mu^\dagger$, and φ_μ, it is shown that ϕ_μ, $\phi_\mu^\dagger$, and φ_μ correspond to the electroweak gauge bosons W_μ^+, W_μ^-, and W_μ^3, respectively, which are composite particles of preons and antipreons.

1. INTRODUCTION

We have considered a preon model of quarks and leptons in which preons and antipreons construct the $SU(2)$ doublet (Sugita *et al.*, 1991, 1992a,b, 1994a,b; Okamoto *et al.*, 1992a,b, 1995). It was shown that CP is violated in only the first family of quarks and leptons.

To account for CP violation in the first family, we have considered a preon $\langle\langle a\rangle\rangle$ with spin 1/2 and electric charge 1/2, a preon $\langle\langle l\rangle\rangle$ with spin 1/2 and electric charge -1, and a preon $\langle\langle q_i\rangle\rangle$ with spin 1/2 and electric charge $-1/3$ having colors of $i = R, G, B$. Then the first family of leptons and quarks is written as

$$\begin{pmatrix} \nu_L \\ e_L \end{pmatrix} = \begin{pmatrix} a_L \\ a_L^{cp} \end{pmatrix} a_R l_L, \qquad e_R = (a_R^{cp} a_R l_R)$$

$$\begin{pmatrix} u_{iL} \\ d_{iL} \end{pmatrix} = \begin{pmatrix} a_L \\ a_L^{cp} \end{pmatrix} a_R q_{iL}, \qquad u_{iR} = (a_R a_R q_{iR}), \qquad d_{iR} = (a_R^{cp} a_R q_{iR})$$

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where the subscripts L and R denote left- and right-handed particles, respectively, and a_L^{cp} means the left-handed particle operated on by charge conjugation C and then parity transformation P, namely

$$a_L^{cp} \equiv \gamma^0 C \gamma^0 \tfrac{1}{2}(1 - \gamma_5)a^* \tag{1}$$

From this model, we can account for CP violation. The Lagrangian describing the electroweak interactions is written as

$$L = \bar{\chi}\gamma^{\mu}\left(i\partial_{\mu} - \frac{g}{2}\tau \cdot W_{\mu}\right)\chi \tag{2}$$

where χ is the weak-isospin doublet, g is the coupling constant, $\mathbf{W}_{\mu}$ are three gauge fields of $SU(2)_L$, and $\tau/2$ are generators of $SU(2)_L$.

Now consider the following weak isospin doublet:

$$\chi = (a_L \, A a_L^{cp})^T \tag{3}$$

where the superscript T means transposed, and A is a matrix. To satisfy the $SU(2)_L$ gauge symmetry, the matrix A must satisfy the condition (Sugita *et al.*, 1994a)

$$ACA^*C = -I_4 \tag{4}$$

where I_4 is a 4×4 unit matrix. If $A = M\gamma^0$ and M is a Lorentz scalar, equation (2) is invariant under the Lorentz transformation. Then using M, we can rewrite equation (4) as

$$\tilde{M}M^* = -I_4 \tag{5}$$

where $\tilde{M}$ means that γ_5, γ^{μ}, and $\sigma^{\mu\nu}$ are replaced by $-\gamma_5$, $-\gamma^{\mu *}$, and $-\sigma^{\mu\nu *}$, respectively, in M. Here $\sigma^{\mu\nu}$ is defined as $(i/2)[\gamma^{\mu}, \gamma^{\nu}]$. If the matrix M satisfies (5), the $SU(2)$ and the Lorentz invariances hold. For example, we can choose $\exp[id_{\mu}\gamma^{\mu}]\gamma_5$, $\exp[id_{\mu\nu}\gamma_5\sigma^{\mu\nu}]\gamma_5$, and $\exp[it_{\kappa\mu\nu}\gamma_5\sigma^{\kappa\mu}\gamma^{\nu}]\gamma_5$ as the matrix M. Here d_{μ} is a real vector in Minkowski space and independent of space.

In a previous paper (Okamoto *et al.*, 1995), as an example, we considered the matrix

$$\begin{aligned} A &= M\gamma^0 \\ &= \exp[id_{\mu}\gamma^{\mu}]\gamma_5\gamma^0 \end{aligned} \tag{6}$$

and showed direct CP violation in β-decay of the nucleon satisfying both the $SU(2)$ and Lorentz invariances. That is, Hermite conjugation of the preon tensor $T_{\mu\nu}^{\langle\text{preon}\rangle\dagger}$ is written as

$$T_{\mu\nu}^{\langle\text{preon}\rangle\dagger} = \tfrac{1}{2}d^{-2}\sin^2 d \, \mathrm{Tr}(K^{\dagger} d_{\sigma}\gamma^{\sigma\dagger}\gamma^0\gamma_{\mu}K'\gamma_{\nu}d_{\rho}\gamma^{\rho}\gamma^0\tfrac{1}{2}(1 - \gamma_5)) \tag{7}$$

On the other hand, $T_{\mu\nu}^{\langle\text{preon}\rangle cp}$ is written as

$$T_{\mu\nu}^{\langle\text{preon}\rangle cp} = \tfrac{1}{2}d^{-2}\sin^2 d\,\text{Tr}(K^\dagger d_\sigma \gamma^\sigma \gamma^0 \gamma_\mu K' \gamma_\nu d_\rho \gamma^{\rho\dagger}\gamma^0 \tfrac{1}{2}(1 - \gamma_5)) \tag{8}$$

The difference between (7) and (8) is that the dagger † operates on γ_σ and γ_ρ. Thus, we obtain $T_{\mu\nu}^{\langle\text{preon}\rangle\dagger} \neq T_{\mu\nu}^{\langle\text{preon}\rangle cp}$ and we can show direct CP violation in β-decay of the nucleon satisfying both the $SU(2)$ and Lorentz invariances.

Thus, based on the preon model, we could explain CP violation in the first family of quarks and leptons by considering β-decay of the nucleon. In the following, we consider the gauge bosons composed of preons and antipreons by introducing auxiliary fields.

2. COMPOSITE MODEL OF GAUGE BOSONS

The preon a_L and antipreon a_L^{cp} contribute to the $SU(2)$ doublet in electroweak interactions. Therefore, the gauge bosons W_μ^+, W_μ^-, and W_μ^3 derived from the $SU(2)$ gauge symmetry are constructed from the preon a_L and antipreon a_L^{cp}.

Consider the simple Lagrangian describing the preon fields,

$$L = L_{\text{free}} + L_{\text{int}} \tag{9}$$

where

$$L_{\text{free}} = \overline{a_L}\gamma^\mu i\partial_\mu a_L + \overline{a_L^{cp}}\gamma^\mu i\partial_\mu a_L^{cp}$$

$$L_{\text{int}} = \lambda\{(\overline{a_L}\gamma^\mu a_L)(\overline{a_L^{cp}}\gamma_\mu a_L^{cp}) + (\overline{a_L^{cp}}\gamma^\mu a_L)(\overline{a_L}\gamma_\mu a_L^{cp})\} \tag{10}$$

Here λ is a coupling constant of four-Fermi interactions. This Lagrangian is fundamental, since it is constructed from the preon field a_L and antipreon field a_L^{cp}. For simplicity, we consider $A = 1$ in (3). The following calculation is valid for the case including A. The generating functional Z without external fields is represented by the path integral by

$$Z = \int Da^{cp}\, Da \exp i \int d^4x\, [L_{\text{free}} + L_{\text{int}}] \tag{11}$$

Introducing the auxiliary fields ϕ_μ, $\phi_\mu^\dagger$, φ_μ, and φ_μ^{cp}, we rewrite the term L_{int} in equation (11) as

$$\begin{aligned}
&\exp i \int d^4x\, L_{\text{int}} \\
&= \frac{1}{K}\int D\phi^\dagger\, D\phi \exp i \int d^4x\, [-m^2\phi_\mu^\dagger\phi^\mu] \\
&\quad \times \frac{1}{N}\int D\varphi^{cp}\, D\varphi \exp i \int d^4x\, [-\mu^2\varphi_\mu^{cp}\varphi^\mu] \\
&\quad \times \exp i \int d^4x\, \lambda\{(\overline{a_L}\gamma^\mu a_L)(\overline{a_L^{cp}}\gamma_\mu a_L^{cp}) + (\overline{a_L^{cp}}\gamma^\mu a_L)(\overline{a_L}\gamma_\mu a_L^{cp})\}
\end{aligned} \tag{12}$$

where

$$K \equiv \int D\phi^{\dagger}\, D\phi \,\exp i \int d^4x\, [-m^2\phi_{\mu}^{\dagger}\phi^{\mu}]$$

$$N \equiv \int D\varphi^{cp}\, D\varphi \,\exp i \int d^4x\, [-\mu^2\varphi_{\mu}^{cp}\varphi^{\mu}] \tag{13}$$

Furthermore, we carry out the transformation of integral variables in the form

$$\phi'_{\mu} = \phi_{\mu} - \frac{\sqrt{\lambda}}{m}(\overline{a_L^{cp}}\gamma_{\mu}a_L), \qquad \phi'^{\dagger}_{\mu} = \phi^{\dagger}_{\mu} - \frac{\sqrt{\lambda}}{m}(\overline{a_L}\gamma_{\mu}a_L^{cp}) \tag{14}$$
$$\varphi'_{\mu} = \varphi_{\mu} - \frac{\sqrt{\lambda}}{\mu}(\overline{a_L}\gamma_{\mu}a_L), \qquad \varphi'^{cp}_{\mu} = \varphi^{cp}_{\mu} - \frac{\sqrt{\lambda}}{\mu}(\overline{a_L^{cp}}\gamma_{\mu}a_L^{cp})$$

Here the path integral measure is invariant under this transformation. Thus we obtain

$$\exp i \int d^4x\, L_{\text{int}} = \frac{1}{K\cdot N}\int D\phi^{\dagger}\, D\phi\, D\varphi^{cp}\, D\varphi \,\exp i \int d^4x\, L_{A\text{int}} \tag{15}$$

where

$$\begin{aligned} L_{A\text{int}} = &-m^2\left\{\phi^{\dagger}_{\mu} - \frac{\sqrt{\lambda}}{m}(\overline{a_L}\gamma_{\mu}a_L^{cp})\right\}\left\{\phi^{\mu} - \frac{\sqrt{\lambda}}{m}(\overline{a_L^{cp}}\gamma^{\mu}a_L)\right\} \\ &- \mu^2\left\{\varphi^{cp}_{\mu} - \frac{\sqrt{\lambda}}{\mu}(\overline{a_L^{cp}}\gamma_{\mu}a_L^{cp})\right\}\left\{\varphi^{\mu} - \frac{\sqrt{\lambda}}{\mu}(\overline{a_L}\gamma^{\mu}a_L)\right\} \\ &+ \lambda\{(\overline{a_L}\gamma^{\mu}a_L)(\overline{a_L^{cp}}\gamma_{\mu}a_L^{cp}) + (\overline{a_L^{cp}}\gamma^{\mu}a_L)(\overline{a_L}\gamma_{\mu}a_L^{cp})\} \\ = &-m^2\phi^{\dagger}_{\mu}\phi^{\mu} - \mu^2\varphi^{cp}_{\mu}\varphi^{\mu} + m\sqrt{\lambda}\{(\overline{a_L}\gamma_{\mu}a_L^{cp})\phi^{\mu} + \phi^{\dagger}_{\mu}(\overline{a_L^{cp}}\gamma^{\mu}a_L)\} \\ &+ \mu\sqrt{\lambda}\{(\overline{a_L^{cp}}\gamma_{\mu}a_L^{cp})\varphi^{\mu} + \varphi^{cp}_{\mu}(\overline{a_L}\gamma^{\mu}a_L)\} \end{aligned} \tag{16}$$

From the above, the generating functional is written as

$$Z = \frac{1}{K\cdot N}\int Da^{cp}\, Da\, D\phi^{\dagger}\, D\phi\, D\varphi^{cp}\, D\varphi \,\exp i \int d^4x\, L_A \tag{17}$$

where the Lagrangian L_A includes all auxiliary fields and is written as

$$\begin{aligned} L_A = &\ \overline{a_L}\gamma^{\mu}i\partial_{\mu}a_L + \overline{a_L^{cp}}\gamma^{\mu}i\partial_{\mu}a_L^{cp} - m^2\phi^{\dagger}_{\mu}\phi^{\mu} - \mu^2\varphi^{cp}_{\mu}\varphi^{\mu} \\ &+ m\sqrt{\lambda}\{(\overline{a_L}\gamma_{\mu}a_L^{cp})\phi^{\mu} + \phi^{\dagger}_{\mu}(\overline{a_L^{cp}}\gamma^{\mu}a_L)\} \\ &+ \mu\sqrt{\lambda}\{(\overline{a_L^{cp}}\gamma_{\mu}a_L^{cp})\varphi^{\mu} + \varphi^{cp}_{\mu}(\overline{a_L}\gamma^{\mu}{}_L)\} \end{aligned} \tag{18}$$

From this, we average each auxiliary field. Thus we obtain $\langle\phi_\mu\rangle$ as

$$\begin{aligned}
\langle\phi_\mu\rangle &= \frac{1}{K\cdot N}\int Da^{cp}\,Da\,D\phi^\dagger\,D\phi\,D\varphi^{cp}\,D\varphi\;\phi_\mu \exp i\int d^4x\,L_A \\
&= \frac{1}{K\cdot N}\int Da^{cp}\,Da\,D\phi'^\dagger\,D\phi'\,D\varphi'^{cp}\,D\varphi'\left\{\phi'_\mu + \frac{\sqrt{\lambda}}{m}(\overline{a_L^{cp}}\gamma^\mu a_L)\right\} \\
&\quad\times exp\; i\int d^4x\,L_A \\
&= \frac{1}{K\cdot N}\int Da^{cp}\,Da\,D\phi'^\dagger\,D\phi'\,D\varphi'^{cp}\,D\varphi' \\
&\quad\times \exp\left[i\int d^4x\,(-m^2\phi'^\dagger_\mu\phi'^\mu - \mu^2\varphi'^{cp}_\mu\varphi'^\mu)\right] \\
&\quad\times \frac{\sqrt{\lambda}}{m}(\overline{a_L^{cp}}\gamma_\mu a_L)\exp\left[i\int d^4x\,(L_{\text{free}} + L_{\text{int}})\right] \\
&= \frac{\sqrt{\lambda}}{m}\int Da^{cp}\,Da\,(\overline{a_L^{cp}}\gamma_\mu a_L)\exp i\int d^4x\,[L_{\text{free}} + L_{\text{int}}] \\
&= \frac{\sqrt{\lambda}}{m}\langle(\overline{a_L^{cp}}\gamma_\mu a_L)\rangle
\end{aligned} \tag{19}$$

Similarly, we obtain

$$\langle\phi^\dagger_\mu\rangle = \frac{\sqrt{\lambda}}{m}\langle(\overline{a_L}\gamma_\mu a_L^{cp})\rangle \tag{20}$$

$$\langle\varphi_\mu\rangle = \frac{\sqrt{\lambda}}{\mu}\langle(\overline{a_L}\gamma_\mu a_L)\rangle \tag{21}$$

$$\langle\varphi^{cp}_\mu\rangle = \frac{\sqrt{\lambda}}{\mu}\langle(\overline{a_L^{cp}}\gamma_\mu a_L^{cp})\rangle \tag{22}$$

Thus, we can see that each auxiliary field is a composite field of preons. Now, we shall examine the interaction term in the Lagrangian L_A. First, the term $m\sqrt{\lambda}(\overline{a_L}\gamma^\mu a_L^{cp})\phi_\mu$ is interpreted as the interaction term describing the phenomenon in which the preon a_L^{cp} absorbs the auxiliary vector field ϕ_μ at the coupling constant $m\sqrt{\lambda}$ and changes to the preon a_L. Thus, ϕ_μ corresponds to a charged gauge boson W^+_μ in the preon model. Similarly, $\phi^\dagger_\mu$ corresponds to a charged gauge boson W^-_μ. Next, the term $\mu\sqrt{\lambda}(\overline{a_L^{cp}}\gamma^\mu a_L^{cp})\varphi_\mu$ means that the preon a_L^{cp} absorbs the auxiliary vector field φ_μ at a coupling constant

$\mu\sqrt{\lambda}$. Thus, φ_μ corresponds to a neutral gauge boson W^3_μ in the preon model. From the definition, we obtain $\varphi^{cp}_\mu = -\varphi^\mu$. Thus, φ^{cp}_μ corresponds to a neutral gauge boson $-W^{3\mu}$.

Finally, it is concluded that the auxiliary vector fields ϕ_μ, $\phi^\dagger_\mu$, and φ_μ representing the composite preon field correspond to the gauge bosons W^+_μ, W^-_μ, and W^3_μ, respectively, in electroweak interactions. This is summarized as follows:

$$\langle W^+_\mu \rangle = \frac{\sqrt{\lambda}}{m} \langle (\overline{a^{cp}_L} \gamma_\mu a_L) \rangle \tag{23}$$

$$\langle W^-_\mu \rangle = \frac{\sqrt{\lambda}}{m} \langle (\overline{a_L} \gamma_\mu a^{cp}_L) \rangle \tag{24}$$

$$\langle W^3_\mu \rangle = \frac{\sqrt{\lambda}}{\mu} \langle (\overline{a_L} \gamma_\mu a_L) \rangle \tag{25}$$

3. $SU(2)$ GAUGE SYMMETRY

In the following, we shall show that the term L_{A-M} minus the mass term in the Lagrangian L_A has exactly the $SU(2)$ gauge symmetry. The Lagrangian L_{A-M} is written as

$$\begin{aligned} L_{A-M} &= \overline{a_L} \gamma^\mu i\partial_\mu a_L + \overline{a^{cp}_L} \gamma^\mu i\partial_\mu a^{cp}_L \\ &\quad + m\sqrt{\lambda}\{(\overline{a_L}\gamma_\mu a^{cp}_L)\phi^\mu + \phi^\dagger_\mu(\overline{a^{cp}_L}\gamma^\mu a_L)\} \\ &\quad + \mu\sqrt{\lambda}\{(\overline{a^{cp}_L}\gamma_\mu a^{cp}_L)\varphi^\mu + \varphi^{cp}_\mu(\overline{a_L}\gamma^\mu a_L)\} \\ &= \overline{a_L} \gamma^\mu i\partial_\mu a_L + \overline{a^{cp}_L} \gamma^\mu i\partial_\mu a^{cp}_L \\ &\quad + m\sqrt{\lambda}\{(\overline{a_L}\gamma_\mu a^{cp}_L)W^{+\mu} + W^-_\mu(\overline{a^{cp}_L}\gamma^\mu a_L)\} \\ &\quad + \mu\sqrt{\lambda}\{(\overline{a^{cp}_L}\gamma_\mu a^{cp}_L)W^{3\mu} - W^{3\mu}(\overline{a_L}\gamma_\mu a_L)\} \end{aligned} \tag{26}$$

Here, replacing $-W^{\pm\mu}$ by $W^{\pm\mu}$ and putting

$$m\sqrt{\lambda} = \frac{g}{\sqrt{2}}, \qquad \mu\sqrt{\lambda} = \frac{g}{2} \tag{27}$$

we obtain

$$\begin{aligned} L_{A-M} &= \overline{a_L} \gamma^\mu i\partial_\mu a_L + \overline{a^{cp}_L} \gamma^\mu i\partial_\mu a^{cp}_L \\ &\quad - \frac{g}{2}\{(\overline{a_L}\gamma_\mu a^{cp}_L)\sqrt{2}W^{+\mu} + \sqrt{2}W^-_\mu(\overline{a^{cp}_L}\gamma^\mu a_L)\} \\ &\quad - \frac{g}{2}\{(\overline{a_L}\gamma_\mu a_L)W^{3\mu} - W^{3\mu}(\overline{a^{cp}_L}\gamma_\mu a^{cp}_L)\} \end{aligned}$$

$$
\begin{aligned}
&= \overline{a_L}\gamma^\mu i\partial_\mu a_L + \overline{a_L^{cp}}\gamma^\mu i\partial_\mu a_L^{cp} \\
&\quad - \frac{g}{2}\{\overline{a_L}\gamma^\mu(a_L W_\mu^3 + a_L^{cp}\sqrt{2}W_\mu^+) + \overline{a_L^{cp}}\gamma^\mu(a_L\sqrt{2}W_\mu^- - a_L^{cp}W_\mu^3)\} \\
&= \overline{a_L}\gamma^\mu i\partial_\mu a_L + \overline{a_L^{cp}}\gamma^\mu i\partial_\mu a_L^{cp} \\
&\quad - \frac{g}{2}(\overline{a_L}\ \overline{a_L^{cp}})\gamma^\mu \begin{pmatrix} W_\mu^3 & \sqrt{2}W_\mu^+ \\ \sqrt{2}W_\mu^- & -W_\mu^3 \end{pmatrix} \begin{pmatrix} a_L \\ a_L^{cp} \end{pmatrix}
\end{aligned}
\tag{28}
$$

Here we put $W_\mu^\pm = (1/\sqrt{2})(W_\mu^1 \mp iW_\mu^2)$ and $\chi = (a_L\, a_L^{cp})^T$. Finally, we obtain

$$
L_{A-M} = \overline{\chi}\gamma^\mu\left(i\partial_\mu - \frac{g}{2}\boldsymbol{\tau}\cdot\mathbf{W}_\mu\right)\chi \tag{29}
$$

Equation (29) is quite the same as equation (2). Therefore, equation (29) satisfies the $SU(2)$ gauge symmetry.

From equations (23)–(25), we considered that the electroweak gauge bosons are composite fields of preons and antipreons. Then we showed that the electroweak interaction Lagrangian (29) satisfying the $SU(2)$ gauge symmetry is derived from the fundamental Lagrangian (9) describing only the preon and antipreon fields. This shows the validity of the composite model of the gauge bosons.

4. CONCLUSIONS

We have considered the fundamental Lagrangian describing the preon and antipreon fields. Furthermore, we have considered a composite model of electroweak gauge bosons using the auxiliary field method. We concluded that the auxiliary vector fields representing the preon fields ϕ_μ, $\phi_\mu^\dagger$, and φ_μ correspond to the electroweak gauge bosons W_μ^+, W_μ^-, and W_μ^3, respectively.

From the interaction Lagrangian for the gauge bosons and the preons, we obtain the electroweak interaction Lagrangian which is invariant under the $SU(2)$ gauge transformation. This shows the validity of the composite model of the gauge bosons constructed by the preon and antipreon.

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